Introducing the Busy Beaver Problem

Turing Machine

Busy Beaver Function

$$|\ TM_s \quad \text{with} \ n \ \text{states}\ | < \infty$$

Halt $\longleftarrow$ $\longrightarrow$ Never Halt

$$\{\ TM_1, TM_2, -, TM_N\}$$

TM takes longest to halt $\quad BB_n$
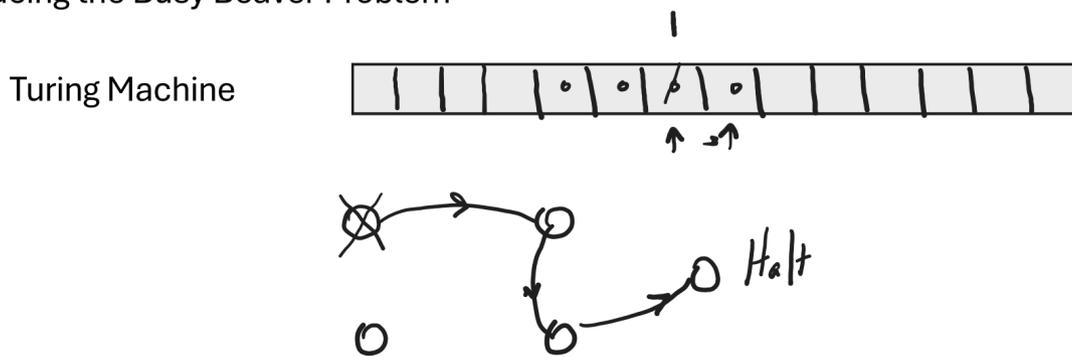
$$\boxed{BB(n)}$$
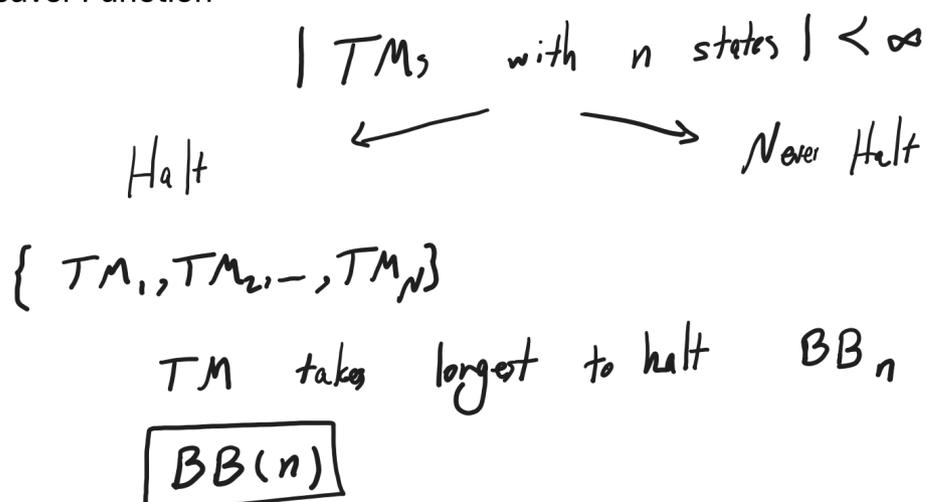
Known Busy Beaver Values

n	$BB(n)$
1	1
2	6
3	21
4	107
5	47 M
6	$1.0 \uparrow\uparrow 15 < 10^{10^{10^{\cdots^{10}}}}$

Busy Beaver is Uncomputable

$$n \to BB(n)$$

Computing BB(n) => Solving the Halting Problem

$n \to \boxed{BB} \to \underline{BB(n)}$ \quad Run TM for BB(n) steps.

TM Halt?

$\hookrightarrow$ # states = n